

ON DOMINATING SOMBOR ENERGY OF A GRAPH

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ABSTRACT. In graph theory, a set of vertices $D \subseteq V(G)$ in a simple graph G is termed a dominating set if every vertex not in D is adjacent to at least one member of D , formally expressed as $N[D] = V(G)$. This work bridges these two concepts such as domination and topological indices. We establish several bounds for this new graph invariant. The paper concludes by demonstrating potential applications of new dominating energy in the field of chemistry.

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1. Introduction

All graphs considered in this paper are finite, simple and undirected. In particular, these graphs do not possess loops. Let $G = (V, E)$ be a graph with the vertex set $V(G) = \{v_1, v_2, v_3, \dots, v_n\}$ and the edge set $E(G) = \{e_1, e_2, e_3, \dots, e_m\}$, that is $|V(G)| = n$ and $|E(G)| = m$. The vertex u and v are adjacent if $uv \in E(G)$. The open(closed) neighborhood of a vertex $v \in V(G)$ is $N(v) = \{u : uv \in E(G)\}$ and $N[v] = N(v) \cup \{v\}$ respectively. The degree of a vertex $v \in V(G)$ is denoted by $d_G(v)$ and is defined as $d_G(v) = |N(v)|$. A vertex $v \in V(G)$ is pendant if $|N(v)| = 1$ and is called support vertex if it is adjacent to pendant vertex. Any vertex $v \in V(G)$ with $|N(v)| > 1$ is called internal vertex. If $d_G(v) = r$ for every vertex $v \in V(G)$, where $r \in \mathbb{Z}^+$ then G is called r -regular. If $r = 2$ then it is called cycle graph C_n and for $r = 3$ it is called the cubic graph. A graph G is unicyclic if $|V| = |E|$. A graph G is called a block graph, if every block in G is a complete graph. For undefined terminologies we refer the reader to [15].

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Molecular descriptors give hope that the journey throughout endless chemical space won't be a random wandering but a methodical voyage toward substances of importance to mankind. Nowadays, there is a myriad of molecular descriptors, and among them, the topological indices have a prominent place. Topological index is simply a numeric associated with the molecular graph. So far, large number of such quantities are put forward by many researchers right from 1972[6]. An useful topological index is one which has a good predicting power in QSPR studies. Therefore, topological indices can be categorized into two categories useful and not so useful TI's. One of the most useful topological index is the Sombor index $SO(G)$ which is put forward by I Gutman[4]:

$$SO(G) = \sum_{uv \in E(G)} [\sqrt{\deg(u)^2 + \deg(v)^2}] \quad (1)$$

Boris Furtula and Ivan Gutman[2] have put forward a degree based topological indices viz., a forgotten topological index which is defined as

$$F(G) = \sum_{u \in V(G)} d_G(u)^3 = \sum_{uv \in E(G)} [d_G(u)^2 + d_G(v)^2].$$

A set $S \subseteq V$ is a *dominating set* of G if each vertex in $V - S$ is adjacent to some vertex in S . The *domination number* $\gamma_{so}(G)$ is the smallest cardinality of a dominating set. A dominating set is said to be minimal, if no proper subset of S is a dominating set of G . It is well known that, a maximal independent set of G is a minimal dominating set of G . An excellent treatment of the fundamentals of domination is given in the book by Haynes et al. [9]. A survey of several advanced topics in domination is given in the book edited by Haynes et al. [10]. Various types of domination have been defined and studied by several authors and more than 75 models of domination are listed in the appendix of Haynes et al.[8].

The energy $E(G)$ of a graph G is equal to the sum of the absolute values of the eigenvalues of the adjacency matrix of G . This quantity, introduced almost 30 years ago [12] and having a clear connection to chemical problems [14], has in newer times attracted much attention of mathematicians and mathematical chemists [8–11, 19, 21, 22, 26, 28].

In connection with energy (that is defined in terms of the eigenvalues of the adjacency matrix), energy-like quantities were considered also for the other matrices: Laplacian [14], distance [16], incidence [17], minimum covering energy [1] etc. Recall that a great variety of matrices has so far been associated with graphs [4, 5, 10, 27].

Motivated by Sombor index and domination number, here we define the minimum Sombor dominating matrix abbreviated as (S-dominating matrix). The S-dominating matrix of G is the $n \times n$ matrix defined by $A_S(G) = (a_{ij})$, where

$$a_{ij} = \begin{cases} \sqrt{\deg(v_i)^2 + \deg(v_j)^2}, & \text{if } v_i v_j \in E; \\ 1, & \text{if } i = j \text{ and } v_i \in S; \\ 0, & \text{otherwise.} \end{cases}$$

The characteristic polynomial of $A_S(G)$ is denoted by $f_n(G, \lambda) := \det(\lambda I - A_S(G))$.

The S-dominating eigenvalues of a graph G are the eigenvalues of $A_S(G)$. Since $A_S(G)$ is real and symmetric, its eigenvalues are real numbers and we label them in non-increasing order $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$. The S-dominating energy of G is then defined as

$$E_S(G) = \sum_{i=1}^n |\lambda_i|.$$

Note. The Somber dominating matrix is similar to (but not identical with) the Sombor matrix of a graph with self-loops [2].

2. Results

We begin with the following straightforward observations. Note that the trace of $A_S(G) = \gamma(G)$.

Let $G = (V, E)$ be a graph with Somber dominating set S . Let $f_n(G, \lambda) = c_0 \lambda^n + c_1 \lambda^{n-1} + \dots + c_n$ be the characteristic polynomial of $A_S(G)$. Then

- (1) $c_0 = 1$,
- (2) $c_1 = -|D| = -\gamma_{so}(G)$.

Theorem 2.1. *If $\lambda_1, \lambda_2, \dots, \lambda_n$ are the eigenvalues of $A_S(G)$, then*

$$\sum_{i=1}^n \lambda_i^2 = 2F(G) + \gamma_{so}(G).$$

We now obtain bounds for $E_S(G)$ of G , similar to McClelland's inequalities [20] for graph energy.

Theorem 2.2. *Let G be a graph of order n and size m with $\gamma_{so}(G) = k$. Then*

$$E_S(G) \leq \sqrt{n(2m + k)}. \quad (2)$$

Proof. Let $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$ be the eigenvalues of $A_S(G)$. Bearing in mind the Cauchy-Schwarz inequality,

$$\left(\sum_{i=1}^n a_i b_i \right)^2 \leq \left(\sum_{i=1}^n a_i \right)^2 \left(\sum_{i=1}^n b_i \right)^2$$

we choose $a_i = 1$ and $b_i = |\lambda_i|$, which by Theorem 3 implies

$$E_S^2 = \left(\sum_{i=1}^n |\lambda_i| \right)^2$$

$$\begin{aligned} &\leq n \left(\sum_{i=1}^n |\lambda_i|^2 \right) \\ &= n \sum_{i=1}^n \lambda_i^2 \\ &= 2(2m + k). \end{aligned}$$

□

Theorem 2.3. *Let G be a graph of order n and size m with $\gamma_{so}(G) = k$. Let $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$ be a non-increasing arrangement of eigenvalues of $A_S(G)$. Then*

$$E_S(G) \geq \sqrt{2mn + nk - \alpha(n)(|\lambda_1| - |\lambda_n|)^2} \tag{3}$$

where $\alpha(n) = n[\frac{n}{2}](1 - \frac{1}{n}[\frac{n}{2}])$, where $[x]$ denotes the integer part of a real number k .

Proof. Let a_1, a_2, \dots, a_n and b_1, b_2, \dots, b_n be real numbers for which there exist real constants a, b, A and B , so that for each $i, i = 1, 2, \dots, n, a \leq a_i \leq A$ and $b \leq b_i \leq B$. Then the following inequality is valid (see [6]).

$$\left| n \sum_{i=1}^n a_i b_i - \sum_{i=1}^n a_i \sum_{i=1}^n b_i \right| \leq \alpha(n)(A - a)(B - b) \tag{4}$$

where $\alpha(n) = n[\frac{n}{2}](1 - \frac{1}{n}[\frac{n}{2}])$. Equality holds if and only if $a_1 = a_2 = \dots = a_n$ and $b_1 = b_2 = \dots = b_n$.

We choose $a_i := |\lambda_i|, b_i := |\lambda_i|, a = b := |\lambda_n|$ and $A = B := |\lambda_1|, i = 1, 2, \dots, n$, inequality (4) becomes

$$\left| n \sum_{i=1}^n |\lambda_i|^2 - \left(\sum_{i=1}^n |\lambda_i| \right)^2 \right| \leq \alpha(n)(|\lambda_1| - |\lambda_n|)^2 \tag{5}$$

Since $E_{G_c}(G) = \sum_{i=1}^n |\lambda_i|, \sum_{i=1}^n |\lambda_i|^2 = \sum_{i=1}^n |\lambda_i|^2 = 2m+k$ and $E_S(G) \leq \sqrt{n(2m + k)}$, the inequality (5) becomes

$$\begin{aligned} n(2m + k) - (E_S)^2 &\leq \alpha(n)(|\lambda_1| - |\lambda_n|)^2 \\ (E_S)^2 &\geq 2mn + nk - \alpha(n)(|\lambda_1| - |\lambda_n|)^2. \end{aligned}$$

Hence equality holds if and only if $\lambda_1 = \lambda_2 = \dots = \lambda_n$. □

Corollary 2.4. *Let G be a graph of order n and size m with $\gamma_{so}(G) = k$. Let $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$ be a non-increasing arrangement of eigenvalues of $A_S(G)$. Then*

$$E_S(G) \geq \sqrt{2mn + nk - \frac{n^2}{4}(|\lambda_1| - |\lambda_n|)^2} \tag{6}$$

Proof. Since $\alpha(n) = n[\frac{n}{2}](1 - \frac{1}{n}[\frac{n}{2}]) \leq \frac{n^2}{4}$, therefore by (3), result follows. □

Theorem 2.5. Let G be a graph of order n and size m with $\gamma_{so}(G) = k$. Let $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$ be a non-increasing arrangement of eigenvalues of $A_S(G)$. Then

$$E_{G_c}(G) \geq \frac{|\lambda_1||\lambda_2|n + 2m + k}{|\lambda_1| + |\lambda_n|} \quad (7)$$

Proof. Let a_1, a_2, \dots, a_n and b_1, b_2, \dots, b_n be real numbers for which there exist real constants r and R so that for each i , $i = 1, 2, \dots, n$ holds $ra_i \leq b_i \leq Ra_i$. Then the following inequality is valid (see [11]).

$$\sum_{i=1}^n b_i^2 + rR \sum_{i=1}^n a_i^2 \leq (r+R) \sum_{i=1}^n a_i b_i. \quad (8)$$

Equality of (8) holds if and only if, for at least one i , $1 \leq i \leq n$ holds $ra_i = b_i = Ra_i$.

For $b_i := |\lambda_i|$, $a_i := 1$, $r := |\lambda_n|$ and $R := |\lambda_1|$, $i = 1, 2, \dots, n$ inequality (8) becomes

$$\sum_{i=1}^n |\lambda_i|^2 + |\lambda_1||\lambda_n| \sum_{i=1}^n 1 \leq (|\lambda_1| + |\lambda_n|) \sum_{i=1}^n |\lambda_i|. \quad (9)$$

Since $\sum_{i=1}^n |\lambda_i|^2 = \sum_{i=1}^n \lambda_i^2 = 2m + k$, $\sum_{i=1}^n |\lambda_i| = E_S(G)$, from inequality (9),

$$2m + k + |\lambda_1||\lambda_n|n \leq (|\lambda_1| + |\lambda_n|)E_S(G)$$

Hence the result. \square

Theorem 2.6. Let G be a graph of order n and size m with $\gamma_{so}(G) = k$. If $\xi = |\det A_S(G)|$, then

$$E_S(G) \geq \sqrt{2m + k + n(n-1)\xi^{\frac{2}{n}}}. \quad (10)$$

Proof.

$$\begin{aligned} (E_S(G))^2 &= \left(\sum_{i=1}^n |\lambda_i| \right)^2 \\ &= \sum_{i=1}^n |\lambda_i|^2 + \sum_{i \neq j} |\lambda_i||\lambda_j|. \end{aligned}$$

Employing the inequality between the arithmetic and geometric means, we obtain

$$\frac{1}{n(n-1)} \sum_{i \neq j} |\lambda_i||\lambda_j| \geq \left(\prod_{i \neq j} |\lambda_i||\lambda_j| \right)^{\frac{1}{n(n-1)}}.$$

Thus,

$$(E_{D_G})^2 \geq \sum_{i=1}^n |\lambda_i|^2 + n(n-1) \left(\prod_{i \neq j} |\lambda_i||\lambda_j| \right)^{\frac{1}{n(n-1)}}$$

$$\begin{aligned} &\geq \sum_{i=1}^n |\lambda_i|^2 + n(n-1) \left(\prod_{i \neq j} |\lambda_i|^{2(n-1)} \right)^{\frac{1}{n(n-1)}} \\ &= 2m + k + n(n-1) \xi_n^{\frac{2}{n}} \end{aligned}$$

□

Lemma 2.7. *If $\lambda_1(G)$ is the largest minimum connected dominating eigenvalue of $A_S(G)$, then $\lambda_1 \geq \frac{2m + \gamma_{so}(G)}{n}$.*

Proof. Let X be any non-zero vector. Then we have $\lambda_1(A) = \max_{x \neq 0} \left\{ \frac{X'AX}{X'X} \right\}$, see [15]. Therefore, $\lambda_1(A_S(G)) \geq \frac{J'AJ}{J'J} = \frac{2m + \gamma_{so}(G)}{n}$. □

Next, we obtain Koolen and Moulton’s [18] type inequality for $E_S(G)$.

Theorem 2.8. *If G is a graph of order n and size m and $2m + \gamma_C(G) \geq n$, then*

$$E_S(G) \leq \frac{2m + \gamma_{so}(G)}{n} + \sqrt{(n-1) \left[(2m + \gamma_{so}(G)) - \left(\frac{2m + \gamma_{so}(G)}{n} \right)^2 \right]} \tag{11}$$

Proof. Bearing in mind the Cauchy-Schwarz inequality,

$$\left(\sum_{i=1}^n a_i b_i \right)^2 \leq \left(\sum_{i=1}^n a_i \right)^2 \left(\sum_{i=1}^n b_i \right)^2.$$

Put $a_i = 1$ and $b_i = |\lambda_i|$ then

$$\begin{aligned} \left(\sum_{i=2}^n a_i b_i \right)^2 &\leq (n-1) \left(\sum_{i=2}^n b_i \right)^2 \\ (E_S(G) - \lambda_1)^2 &\leq (n-1)(2m + \gamma_{so}(G) - \lambda_1^2) \\ E_S(G) &\leq \lambda_1 + \sqrt{(n-1)(2m + \gamma_{so}(G) - \lambda_1^2)}. \end{aligned}$$

Let

$$f(x) = x + \sqrt{(n-1)(2m + \gamma_{so}(G) - x^2)} \tag{12}$$

For decreasing function

$$\begin{aligned} f'(x) &\leq 0 \\ \Rightarrow 1 - \frac{x(n-1)}{\sqrt{(n-1)(2m + \gamma_{so}(G) - x^2)}} &\leq 0 \\ x &\geq \sqrt{\frac{2m + \gamma_{so}(G)}{n}}. \end{aligned}$$

Since $(2m + k) \geq n$, we have $\sqrt{\frac{2m + \gamma_{so}(G)}{n}} \leq \frac{2m + \gamma_{so}(G)}{n} \leq \lambda_1$. Also $f(\lambda_1) \leq f\left(\frac{2m + \gamma_{so}(G)}{n}\right)$.

$$\text{i.e } E_S(G) \leq f(\lambda_1) \leq f\left(\frac{2m+\gamma_{so}(G)}{n}\right).$$
$$\text{i.e } E_S(G) \leq f\left(\frac{2m+\gamma_{so}(G)}{n}\right)$$

Hence by (12), the result follows. □

3. Chemical Applicability of $E_S(G)$

The boiling point of compound is an important physical property for the design and optimization of the processing engineering in chemical and petrochemical industries. Therefore it is important to develop quantitative structure boiling point models for the estimation of boiling points of untested compounds or selection of reliable experimental boiling points of the compounds. Oxo indicates a double bond between an oxygen and carbon atoms. Oxo chemicals are intermediate and derivative chemical compounds which are characteristically used in chemical and manufacturing processes of paints, plasticizers, coatings, adhesives and lubricant additives.

Data Set

There are 192 oxo chemicals in the data collection, comprising 32 aldehydes, 78 ethers, and 82 ketons (Table 1). Ref[4] is where the calculated and experimental

boiling points are found. Table 2 also contains values of molecular descriptors of 192 oxo compounds.

Sl. No.	Compound Name	Exp	Calcd	GNI	SLI	MSLI	FSLI	MFSLI	SCGI	PCGI	MASCI	SAI	RSI
1	Dimethyl ether	-24.9	-24	4.47	4.47	0.89	8.24	0.48	0.89	0.81	0.33	54.0	5.00
2	Ethyl methyl ether	7.4	8.3	7.3	7.3	1.24	13.9	0.66	1.24	1.06	0.23	62	7.07
3	Methyl propyl ether	38.6	39.3	10.12	10.12	1.6	19.55	0.83	1.6	1.31	0.16	70	10
4	Diethyl ether	34.6	37.1	10.12	10.12	1.6	19.55	0.83	1.6	1.31	0.16	70	10
5	Isopropyl methyl ether	30.8	30.1	10.84	12.16	1.35	32.08	0.56	1.5	1.16	0.22	47.62	80.62
6	Butyl methyl ether	70.3	68.9	12.95	12.95	1.95	25.21	1.01	1.95	1.56	0.11	78	14.14
7	Ethyl propyl ether	63.9	65.3	12.95	12.95	1.95	25.21	1.01	1.95	1.56	0.11	78	14.14
8	Ethyl isopropyl ether	54.1	56.5	13.67	14.99	1.71	37.73	0.74	1.85	1.41	0.15	55.62	228.03
9	Isobutyl methyl ether	58.6	60.2	13.67	14.99	1.71	37.73	0.74	1.85	1.41	0.15	55.62	228.03
10	Sec-butyl methyl ether	59	57.5	13.75	14.84	1.76	36.99	0.79	1.87	1.47	0.14	71.25	205.54
11	Tert-butyl methyl ether	52	50.9	14.97	19.07	1.39	68.7	0.49	1.71	1.22	0.21	44.26	1108.26
12	Methyl pentyl ether	99	97.3	15.78	15.78	2.3	30.87	1.19	2.3	1.81	0.08	86	20
13	Ethyl butyl ether	92.2	92.6	15.78	15.78	2.3	30.87	1.19	2.3	1.81	0.08	86	20
14	Dipropyl ether	89.6	91.2	15.78	15.78	2.3	30.87	1.19	2.3	1.81	0.08	86	20
15	Isopropyl propyl ether	83	82.5	16.5	17.82	2.06	29.33	1.25	2.21	1.66	0.11	63.62	644.98
16	Ethyl isobutyl ether	82	83.9	16.5	17.82	2.06	29.33	1.25	2.21	1.66	0.11	63.62	644.98
17	Isopentyl methyl ether	91	88.7	16.5	17.82	2.06	29.33	1.25	2.21	1.66	0.11	63.62	644.98
18	Methyl 1-methylbutyl ether	91.5	84.8	16.57	17.67	2.11	42.65	0.97	2.22	1.72	0.1	79.25	581.37
19	Methyl 2-methylbutyl ether	93	86.6	16.57	17.67	2.11	42.65	0.97	2.22	1.72	0.1	79.25	581.37
20	Diisopropyl ether	69	73.6	17.21	19.86	1.81	55.91	0.64	2.11	1.51	0.15	41.25	500
21	Methyl tert-pentyl ether	86.3	76.1	17.95	82.66	1.82	73.29	0.73	2.09	1.55	0.13	70	4250
22	1,2-dimethylpropyl methyl ether	81.3	76.3	17.36	19.57	1.9	53.86	0.75	2.14	1.59	0.12	59	360.55
23	2,2-dimethylpropyl methyl ether	80.5	80.9	17.8	21.9	1.75	74.36	0.66	2.06	1.47	0.15	52.26	4701.96
24	1-ethylpropyl methyl ether	88.5	81.9	16.65	17.52	2.17	41.91	1.03	2.24	1.77	0.08	94.88	524.04
25	Ethyl tert-butyl ether	73.1	76	17.8	21.9	1.75	74.36	0.66	2.06	1.47	0.15	52.26	4701.96
26	Hexyl methyl ether	125	124.5	18.61	18.61	2.66	36.53	1.36	2.66	2.06	0.05	94	28.28
27	Ethyl propyl ether	118	118.8	12.95	12.95	1.95	25.21	1.01	1.95	1.56	0.11	78	14.14
28	Butyl propyl ether	117.1	116.4	18.61	18.61	2.66	36.53	1.36	2.66	2.06	0.05	94	28.28
29	Butyl isopropyl ether	107	107.6	19.32	20.65	2.41	49.05	1.09	2.56	1.91	0.07	71.62	1824.28
30	Isobutyl propyl ether	106	107.6	19.32	20.65	2.41	49.05	1.09	2.56	1.91	0.07	71.62	1824.28
31	Ethyl isopentyl ether	111	110	19.32	20.65	2.41	49.05	1.09	2.56	1.91	0.07	71.62	1824.28
32	Tert-butyl propyl ether	100	100.9	20.63	24.73	2.1	80.02	0.84	2.42	1.72	0.1	60.26	19948.75
33	Tert-isobutyl propyl ether	87.6	91.8	21.34	26.77	1.86	92.54	0.57	2.32	1.57	0.14	37.89	15983.61
34	Ethyl 1-ethylbutyl ether	105.5	103.8	19.4	20.5	2.47	48.31	1.15	2.58	1.97	0.07	87.25	1644.38
35	2,2-dimethylpropyl ethyl ether	91.5	103.4	20.63	24.73	2.1	80.02	0.84	2.42	1.72	0.109	60.26	19948.75
36	Ethyl 2-ethyl butyl ether	109.5	104.4	16.5	17.82	2.06	29.33	1.25	2.21	1.66	0.11	63.62	644.98
37	Ethyl tert-pentyl ether	101	99.4	17.95	21.66	1.82	73.29	0.73	2.09	1.55	0.13	70	4250
38	1,2-dimethylpropyl ethyl ether	99.3	102.3	20.19	22.39	2.26	59.52	0.93	2.49	1.84	0.09	67	1019.8
39	Dibutyl ether	140.3	139.8	21.44	21.44	3.01	42.18	1.54	3.01	2.31	0.04	102	40
40	Isopentyl propyl ether	125	131.6	22.15	23.47	2.77	54.71	1.27	2.91	2.16	0.05	79.62	5159.84
41	Butyl isobutyl ether	132	130.6	19.32	20.65	2.41	49.05	1.09	2.56	1.91	0.07	71.62	1824.28
42	Butyl sec-butyl ether	130.5	129.8	22.23	23.33	2.82	53.96	1.32	2.93	2.22	0.05	95.25	4651.02
43	Butyl tert-butyl ether	125	124.9	23.46	27.56	2.45	85.67	1.02	2.77	1.97	0.07	68.26	84635.41
44	Sec-butyl isobutyl ether	122	120.6	22.95	25.36	2.58	66.49	1.05	2.83	2.07	0.06	72.88	3605.55
45	di-sec-butyl ether	121	119.7	23.03	25.21	2.63	65.75	1.11	2.85	2.12	0.06	88.51	3250
46	Diisobutyl ether	122.2	121.3	22.87	25.51	2.52	67.23	0.99	2.82	2.01	0.07	57.25	4000
47	Isobutyl tert-butyl ether	111.5	115.5	24.17	29.59	2.21	98.2	0.74	2.67	1.81	0.1	45.89	67812.75
48	di-tert-butyl ether	107.3	109	25.48	33.68	1.9	129.17	0.49	2.53	1.63	0.14	34052	49130
49	Isopropyl tert-pentyl ether	114.5	113.7	24.32	29.35	2.28	97.13	0.81	2.7	1.9	0.08	63.63	61294.37
50	Hexyl ethyl ether	143	114	21.44	21.44	3.01	42.18	1.54	3.01	2.31	0.041	102	40
51	Heptyl methyl ether	151	150.5	21.44	21.44	3.01	42.18	1.54	3.01	2.31	0.041	102	40
52	1-ethylpropyl propyl ether	128.5	126.5	19.4	20.5	2.47	48.31	1.15	2.58	1.97	0.07	87.25	1655.38
53	Ethyl 1-methylpentyl ether	125	131.9	22.23	23.33	2.82	53.96	1.32	2.93	2.22	0.05	95.25	4651.02
54	Ethyl heptyl ether	166.6	168.3	24.27	24.27	3.36	47.84	1.72	3.36	2.56	0.02	110	56.56
55	Butyl isopentyl ether	157	152.8	24.98	26.3	3.12	60.36	1.44	3.27	2.41	0.03	87.62	14594.24
56	Tert-butyl isopentyl ether	139	137.9	27	32.42	2.56	103.85	0.92	3.03	2.07	0.07	53.89	287705.09
57	Butyl pentyl ether	163	162.5	24.27	24.27	3.36	47.84	1.72	3.36	2.56	0.02	110	56.56
58	Isopropyl isopentyl ether	139	143.7	22.87	25.51	2.52	67.23	0.99	2.82	2.01	0.07	57.25	4000
59	Methyl 1-methylheptyl ether	162	160.7	25.06	26.15	3.17	59.62	1.5	3.28	2.47	0.03	103.25	13155.07
60	Methyl octyl ether	173	175.4	24.27	24.27	3.36	47.84	1.72	3.36	2.56	0.02	110	56.56
61	3,5-dimethylhexyl methyl ether	155.5	155.2	25.78	28.19	2.93	72.14	1.23	3.19	2.32	0.04	80.88	10198.03
62	Ethyl octyl ether	189.2	191.6	3.72	2.81	0.02	53.5	1.89	3.72	2.81	0.02	118	80
63	Ethyl 1,1,3,3-tetramethylbutyl ether	156.5	155.4	28.46	36.26	2.33	133.75	0.73	2.91	1.95	0.08	60.27	188404.94
64	bis(1-ethylpropyl) ether	161.5	159.1	25.93	27.89	3.04	70.66	1.34	3.22	2.42	0.03	112.14	8285.9
65	bis(1-methylbutyl) ether	162	163.3	28.68	30.87	3.34	77.06	1.46	3.56	2.62	0.03	104.51	26000
66	Butyl 1-methylpentyl ether	170	173.7	27.89	28.98	3.53	65.28	1.68	3.64	2.72	0.02	111.25	37208.17
67	diisopentyl ether	172	163	28.52	31.17	3.23	78.54	1.35	3.52	2.51	0.03	73.25	32000
68	dipentyl ether	186.8	183.6	3.72	2.81	0.02	53.5	1.89	3.72	2.81	0.02	118	80
69	Isopropyl heptyl ether	173	176.7	27.81	29.13	3.47	66.02	1.62	3.62	2.66	0.02	95.62	41278.75
70	heptyl propyl ether	187	186.9	3.72	2.81	0.02	53.5	1.89	3.72	2.81	0.02	118	80
71	Isopentyl pentyl ether	174	173.3	27.81	29.13	3.47	66.02	1.62	3.62	2.66	0.02	95.62	41278.75
72	methyl 1-methyloctyl ether	188.5	183.8	27.80	28.98	3.53	65.28	1.68	3.64	2.72	0.02	111.25	37208.17
73	octyl propyl ether	204	208.8	29.92	29.92	4.07	59.15	2.07	4.07	3.06	0.01	126	113.13
74	Isopropyl octyl ether	198.5	197.8	30.64	31.96	3.83	71.68	1.8	3.97	2.91	0.01	103.62	116753.96
75	dihexyl ether	225.7	223.3	33.34	33.34	4.33	64.81	2.25	4.33	3.15	0.01	115	101.19

Sl. No.	Compound Name	Exp	Calcd	GNI	SLI	MSLI	FSLI	MFSLI	SCGI	PCGI	MASCI	SAI	RSI
76	dihetyl ether	258.5	259.2	38.41	38.41	5.13	76.12	2.6	5.13	3.81	0.005	150	320
77	diocetyl ether	291.7	291.6	44.07	44.07	5.84	87.44	2.95	5.84	4.31	0.002	166	640
78	bis(2-ethylhexyl) ether	269.8	268.2	45.81	47.54	5.57	109.52	2.64	5.71	4.22	0.003	183.77	10816000
79	acetaldehyde	20.8	19.	4.47	4.47	0.89	8.24	0.48	0.89	0.81	0.33	54	5
80	propionaldehyde	48.8	50.6	7.3	7.3	1.24	13.9	0.66	1.24	1.06	0.23	62	7.07
81	butyl aldehyde	75.7	78.5	10.12	10.12	1.6	19.55	0.83	1.6	1.31	0.16	70	10
82	pentaldehyde	103	104.5	12.95	12.95	1.95	25.21	1.01	1.95	1.56	0.11	78	14.14
83	2-methyl butanal	92.5	95.8	13.75	14.84	1.76	36.99	0.79	1.87	1.47	0.14	71.25	205.54
84	3-methyl butanal	92.5	97.1	13.67	14.99	1.71	37.73	0.74	1.85	1.41	0.15	55.62	228.03
85	2,2-dimethyl propanal	77.5	79	14.97	19.07	1.39	68.7	0.49	1.71	1.22	0.21	44.26	1108.26
86	Hexanal	128	128.9	15.78	15.78	2.3	30.87	1.19	2.3	1.81	0.08	86	20
87	2-methylpentanal	117	119.6	16.57	17.67	2.11	42.65	0.97	2.22	1.72	0.1	79.25	581.37
88	3-methylpentanal	118	120.6	16.57	17.67	2.11	42.65	0.97	2.22	1.72	0.1	79.25	581.37
89	4-methylpentanal	121	121.6	16.5	17.82	2.06	43.39	0.91	2.21	1.66	0.11	63.62	644.98
90	2-ethylbutanal	117	117.7	16.65	17.52	2.17	41.91	1.03	2.24	1.77	0.08	94.88	524.04
91	heptanal	152.8	151.8	18.61	18.61	2.66	36.53	1.36	2.66	2.06	0.05	94	28.28
92	2-methyl hexanal	141	142.3	19.4	20.5	2.47	48.31	1.15	2.58	1.97	0.07	87.25	1644.38
93	3-methyl hexanal	143	143	16.57	17.67	2.11	42.65	0.97	2.22	1.72	0.1	79.25	581.37
94	4-methyl hexanal	144	143.7	19.4	20.5	2.47	48.31	1.15	2.58	1.97	0.07	87.25	1644.38
95	5-methyl hexanal	143.5	144.2	19.32	20.65	2.41	49.05	1.09	2.56	1.91	0.07	71.62	1824.28
96	2,2-dimethylpentanal	126.5	131.8	20.78	24.49	2.18	78.95	0.9	2.44	1.8	0.09	78.009	18031.22
97	2,3-dimethylpentanal	141	134.1	20.26	22.25	2.31	58.78	0.98	2.51	1.89	0.08	82.63	919.23
98	octanal	171	176.6	21.44	21.44	3.01	42.18	1.54	3.01	2.31	0.04	102	40
99	2-ethylhexanal	160	161.1	22.31	23.18	2.88	53.22	1.38	2.95	2.27	0.04	110.88	4192.37
100	2-propylpentanal	160	160.2	22.31	23.18	2.88	53.22	1.38	2.95	2.27	0.04	110.88	4192.37
101	nonanal	191	194.2	24.27	14.27	3.36	47.84	1.72	3.36	2.56	0.02	110	56.56
102	3,5,5-trimethyl hexanal	170.5	166.7	27.08	32.27	2.62	103.11	0.98	3.04	2.12	0.06	69.52	273361.9
103	decanal	208.5	213.9	27.09	27.09	3.72	53.5	1.89	3.72	2.81	0.02	118	82
104	undecanal	233	232.7	29.92	29.92	4.07	59.15	2.07	4.07	3.06	0.01	126	113.13
105	2-methyl decanal	229	221.7	30.72	31.81	3.88	70.94	1.85	3.99	2.97	0.01	119.25	105240.6
106	dodecanal	254	250.6	32.75	32.75	4.42	64.81	2.25	4.42	3.31	0.01	134	425984
107	2-methylundecanal	246	239	33.55	34.64	4.24	76.59	2.03	4.35	3.22	0.01	127.25	297665.36
108	tridecanal	267	267.8	35.58	35.58	4.78	70.47	2.42	4.78	3.56	0.007	142	226.27
109	tetradecanal	287	284.2	38.41	38.41	5.13	76.12	2.6	5.13	3.81	0.005	150	320
110	pentadecanal	304.9	299.9	41.24	41.24	5.49	81.78	2.78	5.49	4.06	0.003	158	452.54
111	acetone	56.2	47.4	7.93	9.48	0.94	27.16	0.33	1.13	0.86	0.35	24	31.62
112	2-butanone	79.6	75.4	10.84	12.16	1.35	32.08	0.56	1.5	1.16	0.22	47.62	80.62
113	2-pentanone	102	100.4	13.67	14.99	1.71	37.73	0.74	1.85	1.41	0.15	55.62	228.03
114	3-pentanone	101.7	98.3	13.75	14.84	1.76	36.99	0.79	1.87	1.47	0.14	71.25	205.54
115	3-methyl-2-butanone	93.5	92.7	14.45	16.89	1.5	48.94	0.52	1.77	1.29	0.2	35.375	141.42
116	2-hexanone	127	125.2	16.5	17.82	2.06	43.39	0.91	2.21	1.66	0.11	63.62	644.98
117	3-hexanone	123.5	122.2	16.57	17.67	2.11	42.65	0.97	2.22	1.72	0.1	79.25	581.37
118	3-methyl-2-pentanone	118	116	13.67	14.99	1.71	37.73	0.74	1.85	1.41	0.15	55.62	228.03
119	4-methyl-2-pentanone	117	117.4	17.21	19.86	1.81	55.91	0.64	2.11	1.51	0.15	41.25	500
120	2-methyl-3-pentanone	115.5	114.5	17.36	19.57	1.9	53.86	0.75	2.14	1.59	0.12	59.004	360.55
121	3,3-dimethyl-2-butanone	106	107.3	18.65	23.69	1.56	84.56	0.46	1.98	1.35	0.19	32.63	3134.64
122	2-heptanone	151.4	148.5	19.32	20.65	2.41	49.05	1.09	2.56	1.91	0.07	71.62	1824.28
123	3-heptanone	147	145.4	17.4	20.5	2.47	48.31	1.15	2.58	1.97	0.07	87.25	1644.38
124	4-heptanone	144	144.4	19.4	20.5	2.47	48.31	1.15	2.58	1.97	0.07	87.25	1644.38
125	3-methyl-2-hexanone	143.5	139.5	20.19	22.39	2.26	59.52	0.93	2.49	1.84	0.09	67.004	1019.8
126	4-methyl-2-hexanone	139	139.7	20.12	22.53	2.22	60.83	0.87	2.48	1.82	0.09	64.88	1274.75
127	5-methyl-2-hexanone	144	140.5	20.04	22.68	2.17	61.57	0.82	2.46	1.76	0.1	49.25	1414.21
128	2-methyl-3-hexanone	135	134.2	20.19	22.39	2.26	59.52	0.93	2.49	1.84	0.09	67.004	1019.8
129	4-methyl-3-hexanone	134.5	136.4	20.26	22.25	2.31	58.78	0.98	2.51	1.89	0.08	82.63	919.23
130	5-methyl-3-hexanone	135	137.3	20.12	22.53	2.22	60.83	0.87	2.48	1.82	0.09	64.88	1274.75
131	2,2-dimethyl-3-pentanone	125.6	127.4	21.55	26.37	1.96	89.47	0.69	2.35	1.65	0.12	56.26	12636.15
132	2,4-dimethyl-3-pentanone	125.4	128.8	20.97	24.29	2.05	70.73	0.7	2.4	1.71	0.11	46.75	632.45
133	4,4-dimethyl-2-pentanone	126.4	130.6	21.34	26.77	1.86	92.54	0.57	2.32	1.57	0.14	37.89	15983.61
134	2-octanone	172.5	170.7	22.15	23.47	2.77	54.71	1.27	2.91	2.16	0.05	79.62	5159.84
135	3-octanone	167.5	167.6	22.23	23.33	2.82	53.96	1.32	2.93	2.22	0.05	95.25	4651.02
136	4-octanone	165.5	166.1	22.23	23.33	2.82	53.96	1.32	2.93	2.22	0.05	95.25	4651.02
137	2-methyl-4-heptanone	154	157.8	22.95	25.36	2.58	66.49	1.05	2.83	2.07	0.06	72.88	3605.55
138	3-methyl-4-heptanone	153	157.2	23.09	25.07	2.67	64.43	1.16	2.86	2.14	0.05	90.63	2600
139	3-methyl-2-heptanone	164	161.1	23.01	25.22	2.61	65.17	1.1	2.84	2.09	0.06	75	2884.44
140	4-methyl-2-heptanone	160.5	161.6	22.95	25.36	2.58	66.49	1.05	2.83	2.07	0.06	72.88	3605.55
141	5-methyl-2-heptanone	166.5	161.9	22.95	25.36	2.58	66.49	1.05	2.83	2.07	0.06	72.88	3605.55
142	6-methyl-2-heptanone	167	162.2	22.87	25.51	2.52	67.23	0.99	2.82	2.01	0.07	57.25	4000
143	2-methyl-3-heptanone	158	158.3	23.01	25.22	2.61	65.17	1.1	2.84	2.09	0.06	75	2884.44
144	5-methyl-3-heptanone	161	158.6	23.03	25.21	2.63	65.75	1.11	2.85	2.12	0.06	88.51	3250
145	6-methyl-3-heptanone	163.2	159	22.95	25.36	2.58	66.49	1.05	2.83	2.07	0.06	72.88	3605
146	3,3-dimethyl-2-hexanone	151.5	150.3	24.45	29.1	2.36	94.8	0.88	2.72	1.93	0.08	66.37	51000
147	4,4-dimethyl-2-hexanone	154.4	151.3	20.78	24.49	2.18	78.95	0.9	2.44	1.8	0.094	78.009	18031.22
148	5,5-dimethyl-2-hexanone	146	152	24.17	29.59	2.21	98.2	0.74	2.67	1.82	0.1	45.89	67812.74
149	2,2-dimethyl-3-hexanone	146	147.8	24.38	29.2	2.32	95.13	0.87	2.7	1.9	0.08	64.26	53610.67
150	2,4-dimethyl-3-hexanone	145	149.3	20.26	22.25	2.31	58.78	0.98	2.51	1.89	0.08	82.63	919.23
151	2,5-dimethyl-3-hexanone	147.5	149.8	23.73	27.26	2.37	77.7	0.83	2.75	1.94	0.08	52.63	2236.06
152	4,4-dimethyl-3-hexanone	148	147.7	24.53	28.95	2.39	94.06	0.93	2.73	1.98	0.07	82	48457.45
153	4,5-dimethyl-3-hexanone	152	150.4	23.88	26.97	2.46	75.64	0.94	2.77	2.01	0.07	70.37	1612.45
154	5,5-dimethyl-3-hexanone	146.5	148.7	24.25	29.45	2.26	97.46	0.8	2.69	1.87	0.09	61.52	64432.01
155	3-ethyl-3-methyl-2-pentanone	153.8	149.6	24.6	28.86	2.41	93.73	0.94	2.74	2.01	0.07	84.12	46097.92
156	2-ethyl-4-methyl-2-pentanone	154	150.8	23.88	26.97	2.46	75.64	0.94	2.77	2.01	0.07	70.37	1612.45
157	2,2,4-trimethyl-3-pentanone	135.1	139.9	25.16	31.09	2.11	106.34	0.65	2.62	1.78	0.11	44	39650.47
158	2-nonanone	195	191.8	24.98	26.3	3.12	60.36	1.44	3.27	2.41	0.03	87.62	14594.24
159	3-nonanone	190	188.8	25.06	26.15	3.17	59.62	1.5	3.28	2.47	0.03	103.25	13155.07
160	4-nonanone	187.5	187.1	25.06	26.15	3.28	59.62	1.5	3.28	2.47	0.03	103.25	13155.07
161	5-nonanone	188.4	186.5	25.06	26.15	3.17	59.62	1.5	3.28	2.47	0.03	103.25	13155.07
162	4-methyl-2-octanone	184	182.6	25.78	28.19	2.93	72.14	1.23	3.19	2.32	0.04	80.88	10198.03
163	5-methyl-3-octanone	179	179.6	25.85	28.04	2.98	71.4	1.28					

171	7-methyl-4-octanone	178	178	25.78	28.19	2.93	72.14	1.23	3.19	2.32	0.04	80.88	10198.03
172	2,3-dimethyl-4-heptanone	167.5	169.9	26.71	29.8	2.81	81.3	1.11	3.13	2.26	0.05	78.37	4560.7
173	2,6-dimethyl-4-heptanone	169.4	169.9	26.49	30.23	2.69	84.67	0.95	3.09	2.17	0.06	58.51	7905.69
174	3,5-dimethyl-4-heptanone	162	168.8	26.78	29.65	2.86	80.56	1.17	3.14	2.31	0.04	94	4110.96
175	2,2,4,4-tetramethyl-3-pentanone	152	149.2	29.36	37.9	2.17	141.95	0.59	2.83	1.84	0.1	41.26	109858.01
176	2-decanone	210	211.8	27.81	29.13	3.47	66.02	1.62	3.62	2.66	0.02	95.62	41278.75
177	3-decanone	211	209.1	27.89	28.98	3.53	65.28	1.68	3.64	2.72	0.02	111.25	37208.17
178	4-decanone	206.5	207.3	27.89	28.98	3.53	65.28	1.68	3.64	2.72	0.02	111.25	37208.17
179	5-decanone	204	206.4	27.89	28.98	3.53	65.28	1.68	3.64	2.72	0.02	111.25	37208.17
180	2-undecanone	231.5	230.8	30.64	31.96	3.83	71.68	1.8	3.97	2.91	0.01	103.62	116753.96
181	3-undecanone	227	228.4	30.72	31.81	3.88	70.94	1.85	3.99	2.97	0.01	119.25	105240.63
182	5-undecanone	227	225.5	30.72	31.81	3.88	70.94	1.85	3.99	2.97	0.01	119.25	105240.6
183	6-undecanone	226	225.2	30.72	31.81	3.88	70.94	1.85	3.99	2.97	0.01	119.25	105240.6
184	2,2,6,6-tetramethyl-4-heptanone	185	181	34.76	44.05	2.77	157.92	0.8	3.51	2.28	0.06	51.78	2856308.51
185	2-dodecanone	246.5	249	33.47	34.79	4.18	77.33	1.97	4.33	3.16	0.01	111.62	330250.07
186	2-methyl-3-undecanone	233.5	235.1	34.33	36.54	4.03	87.8	1.81	4.26	3.09	0.01	107	184604.22
187	2-tridecanon	263	266.1	36.3	37.62	4.53	82.99	2.15	4.68	3.41	0.009	119.62	934031.7
188	7-tridecanon	261	260.7	36.37	37.47	4.59	82.25	2.21	4.7	3.47	0.088	135.25	841924.8
189	2-methyl-3-tridecanone	267	268	39.99	42.19	4.73	99.12	2.16	4.96	3.59	0.008	123.004	1476833.8
190	7-ethyl-2-methyl-4-undecanone	252.5	255.7	45.06	50.8	4.64	147.43	1.89	5.12	3.63	0.008	130.64	1043598026
191	2-pentadecanone	294	298.7	38.82	39.33	5.006	94.3	2.5	5.06	3.69	0.006	131	2988901.46
192	8-pentadecanone	291	293.4	42.03	43.12	5.3	93.56	2.56	5.41	3.97	0.004	151.25	6795938.43

TABLE 1. Experimental and Calculated BP values and molecular descriptors

The best Linear Correlation for exp is obtained by Sombor dominating energy.

$$exp = +61.001(E_S(G)) + (-28.645) \tag{13}$$

$$n = 192 \quad r = 0.951 \quad F = 1814.004$$

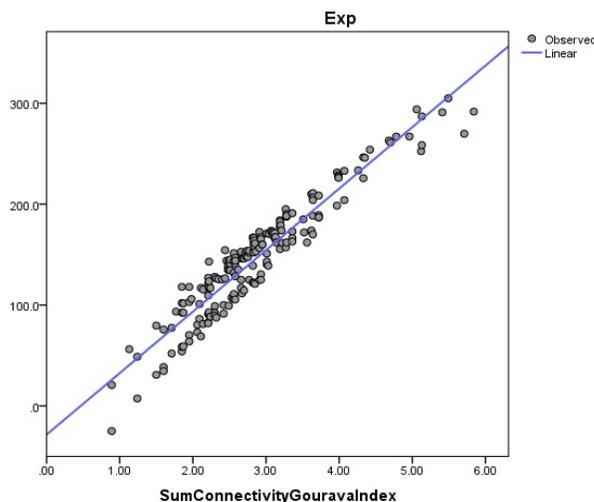


FIGURE 1. linear regression for experimental boiling point

Conflicts of interest : The authors declare no conflict of interest.

Data availability : Not applicable.

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