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ON SOMBOR MATRIX AND ENERGY OF GRAPHS

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Abstract. In this paper, a new variant of Sombor matrix $\mathcal{NS}o(\zeta)$ for a simple graph $\zeta(\mathcal{V},\mathcal{E})$, is defined as if $i \neq j$ its (i,j)-entry is equal to $\sqrt{\varsigma_i^2+\varsigma_j^2}$ and 0 in the either case, where ς_i represents the degree of i^{th} vertex. The Sombor energy $E_{\mathcal{NS}o}(\zeta)$ is the sum of absolute values of eigenvalues of a new variant Sombor matrix. The spectrum and $E_{\mathcal{NS}o}(\zeta)$ has been explored for several well-known classes of graphs, as well as specific graph families and their complements.

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1. Introduction

Let $\zeta(\mathcal{V},\mathcal{E})$ be a simple graph, with $|\mathcal{V}| = \rho$ and $|\mathcal{E}| = \varrho$. The degree of a vertex $v_i \in \mathcal{V}$ is the count of edges connected to v_i denoted by ς_i. The maximum(minimum) vertex and edge degree of a graph is denoted by $\Delta(\delta)$ and $\Delta'(\delta')$ respectively. A graph ζ is said to be complete if $\Delta = \delta = \rho - 1$ denoted by K_ρ. If a vertex set of a graph ζ is partitioned into two sets say $|\mathcal{M}| = \upsilon$ and $|\mathcal{N}| = \nu$ (partite sets) such that every edge meet both $\mathcal{M}$ and $\mathcal{N}$ then the graph is bipartite graph. If every vertex of $\mathcal{M}$ is adjacent to every vertex of $\mathcal{N}$ then the graph is complete bipartite graph denoted as $K_{\upsilon,\nu}$. The graph $K_{1,\rho-1}$ is called as star graph denoted by $\mathcal{S}_\rho$ and the graph $K_{\nu,\nu}$ is called equi-bipartite graph. The complement of a graph ζ denoted by $\overline{\zeta}$ is a graph defined on same vertex set as of ζ such that if two vertices are adjacent in $\overline{\zeta}$, then they are not adjacent in ζ. For more terminologies refer the following [1].

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A square matrix $A(\zeta)$ of order ρ is an adjacency matrix of ζ whose (i, j)-entry is 1 if the vertex v_i is adjacent to v_j, and is 0 otherwise. The energy $E(\zeta)$ of a graph ζ is the sum of the absolute values of eigenvalues of $A(\zeta)$. This quantity is introduced in [2]. Suppose $\kappa_1 \geq \kappa_2 \geq ... \geq \kappa_\rho$ are the eigenvalues of the adjacency matrix $A(\zeta)$ then the energy of the graph ζ is given by

$$E = E(\zeta) = \sum_{i=1}^{\rho} |\kappa_i| \tag{1}$$

In their work, H. S. Ramane et al. [3] introduced the concept of the degree sum matrix for a graph, denoted as $DS(\zeta)$, and derived bounds for the associated degree sum energy. Later, B. Basavangoud et al. [4] extended this idea by proposing the degree square sum matrix and established bounds for the degree square sum energy. Building upon these ideas, a novel variant of the Sombor matrix $\mathcal{NS}o(\zeta)$ is defined as if $i \neq j$ its (i, j)-entry is equal to $\sqrt{\varsigma_i^2 + \varsigma_j^2}$ and 0 in the either case. Here ς_i represents the degree of v_i, for every $i = 1, 2, ..., \rho$.

The characteristic polynomial for $\mathcal{NS}o(\zeta)$ is defined as $\psi(\zeta, \eta) = det(\eta I - \mathcal{NS}o(\zeta)) = |\eta I - \mathcal{NS}o(\zeta)| = \eta^\rho + r_1\eta^{\rho-1} + ... + r_\rho = 0$, here I represents the identity matrix of order ρ. Since $\mathcal{NS}o(\zeta)$ is a real symmetric matrix, the roots of $\psi(\zeta, \eta) = 0$ are real and it can written in descending order as $\eta_1 \geq \eta_2 \geq ... \geq \eta_\rho$, with respective multiplicities $\tau_1, \tau_2, ..., \tau_\rho$ then the spectrum can be written as,

$$Spec(\mathcal{NS}o(\zeta)) = \begin{pmatrix} \eta_1 & \eta_2 & \cdots & \eta_\rho \\ \tau_1 & \tau_2 & \cdots & \tau_\rho \end{pmatrix}$$

where η_1 is the spectral radius. Sombor energy of ζ is given as,

$$E_{\mathcal{NS}o}(\zeta) = \sum_{i=1}^{\rho} |\eta_i| \tag{2}$$

For recent studies on graph energy refer [[3], [4], [5], [6], [7], [8]].

[9] If a, b, c and d are real numbers then the characteristic polynomial of the determinant $\begin{vmatrix} (\lambda + a)I_n - aJ_n & -cJ_{n\times m} \\ -dJ_{m\times n} & (\lambda + b)I_m - bJ_m \end{vmatrix}_{n+m\times n+m}$ is given by $(\lambda + a)^{n-1}(\lambda + b)^{m-1}\{[\lambda - (n-1)a][\lambda - (m-1)b] - nmcd\}$. Also if $\lambda_1, \lambda_2, ..., \lambda_{n+m}$ are eigenvalues of the same determinant then the eigenvalues are given by

$$Spec = \begin{cases} -a & (n-1)times \\ -b & (m-1)times \\ [a(n-1) + b(m-1) \pm \sqrt{[a(n-1) - b(m-1)]^2 + 4nmcd}]/2 & \end{cases}$$

Further, If $(n-1)(m-1)ab \leq nmcd$,

$$\lambda_1 = \sum_{i=2}^{n+m} |\lambda_i| = [a(n-1) + b(m-1) + \sqrt{[a(n-1) - b(m-1)]^2 + 4nmcd}]/2$$

If b and c are two scalars and

$$(A)_{n\times n}=\begin{pmatrix}0 & c & c & \cdots & c\\ c & 0 & b & \cdots & b\\ c & b & 0 & \cdots & b\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ c & b & b & \cdots & 0\end{pmatrix}$$

then,

$$Spec=\begin{cases}-b & (n-1)times\\ [(n-2)b\pm\sqrt{[(n-2)b]^2+4(n-1)c^2}]/2 & \end{cases}$$

Proof. Let b and c be two scalars and

$$(A)_{\rho\times\rho}=\begin{pmatrix}0 & c & c & \cdots & c\\ c & 0 & b & \cdots & b\\ c & b & 0 & \cdots & b\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ c & b & b & \cdots & 0\end{pmatrix}$$

then, by Lemma 1.1 the determinant and characteristic polynomial of matrix A is given by

$$|\lambda I-A|_{n\times n}=\begin{vmatrix}\lambda I_1 & -cJ_{1\times(n-1)}\\ -cJ_{(n-1)\times 1} & (\lambda+b)I_{(n-1)}-bJ_{(n-1)}\end{vmatrix}_{n\times n}$$

$$\begin{aligned}|\lambda I-A| &= (\lambda+0)^{1-1}(\lambda+b)^{n-1-1}\{[\lambda-(1-1)0][\lambda-(n-1-1)b]-(n-1)c^2\}\\ &= (\lambda+b)^{n-2}[\lambda^2-(n-2)b\lambda-(n-1)c^2]\end{aligned}$$

Also,

$$Spec=\begin{cases}-b & (n-1)times\\ [(n-2)b\pm\sqrt{[(n-2)b]^2+4(n-1)c^2}]/2 & \end{cases}$$

□

Let G be simple connected graph then,

$$E_{\mathcal{NS}o}(\zeta) = 2\eta_1$$

where $\eta_1=(n-2)b+\sqrt{[(n-2)b]^2+4(n-1)c^2}$ is the largest eigenvalue.

1.1. Some Families of Graphs. [10] Sunlet graph $\mathcal{S}_\rho$, $\rho \geq 3$ has 2ρ number of vertices.

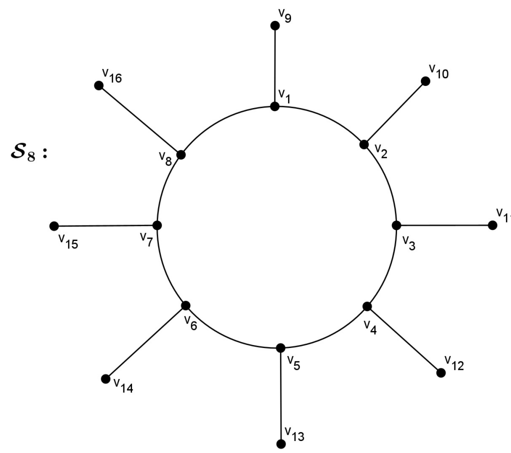

Figure 1: Sunlet graph $\mathcal{S}_8$

[11] Book graph $\mathcal{B}_\rho$, $\rho \geq 1$ has $2\rho + 2$ number of vertices.

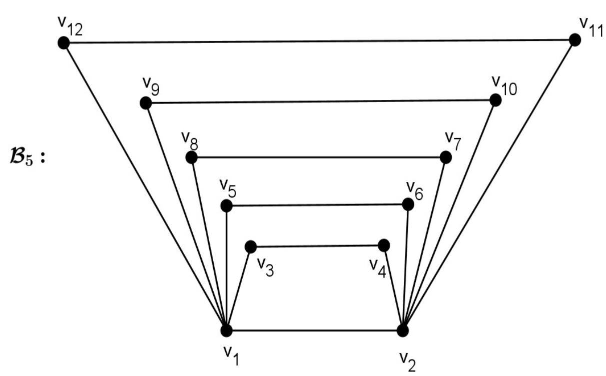

Figure 2: Book graph $\mathcal{B}_5$

[12] Pentagonal snake graph $\mathcal{PS}_\rho$, $\rho \geq 2$ has $4\rho - 3$ number of vertices.

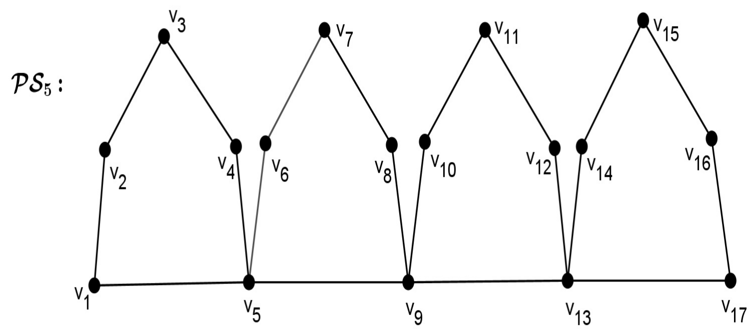

Figure 3: Pentagonal snake graph $\mathcal{PS}_5$

[13] Windmill graph $\mathcal{W}^3_\rho$, $\rho \geq 3$ has $3\rho - 2$ number of vertices.

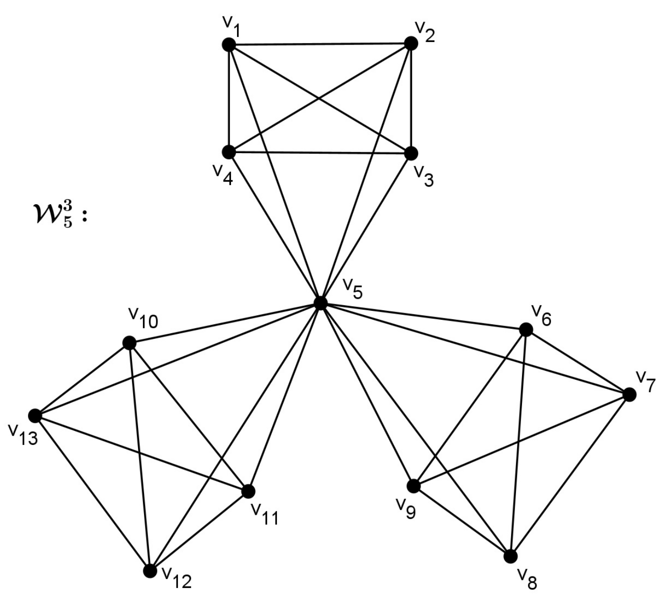

Figure 4: Windmill graph $\mathcal{W}^3_5$

[14] Friendship graph $\mathcal{F}_\rho$, $\rho \geq 1$ has $2\rho + 1$ number of vertices.

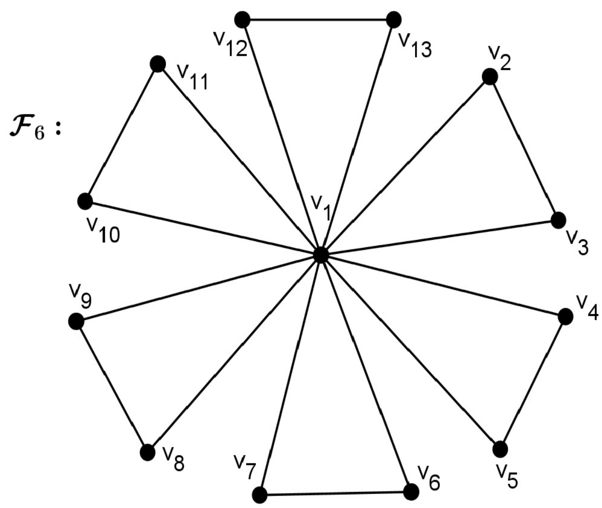

Figure 5: Friendship graph $\mathcal{F}_6$

2. Auxiliary results

- The $E_{\mathcal{NS}o}(K_\rho)$ is $2\sqrt{2}(\rho-1)^2$
- The $E_{\mathcal{NS}o}(\zeta)$ of s-regular graph is $\sqrt{2}s(\rho-1)$
- The $E_{\mathcal{NS}o}(C_\rho)$ is $2\sqrt{2}(\rho-1)$
- The $E_{\mathcal{NS}o}(K_{\rho,\rho})$ is $2\sqrt{2}\rho(2\rho-1)$

3. Main Results

In this section, we obtain spectrum and energy of $\mathcal{NS}o(\zeta)$ for some families of graphs and its complements.

Theorem 3.1. *Let $\mathcal{S}_\rho$, $\rho \geq 3$ be a sunlet graph then*

$$E_{\mathcal{NS}o}(\mathcal{S}_\rho) = 4\sqrt{2}(\rho-1) + 2\sqrt{2[(\rho-1)^2+5\rho^2]}$$

Proof. $\mathcal{S}_\rho$, $\rho \geq 3$ is a sunlet graph with 2ρ vertices then the characteristic polynomial is given as $|\eta I - \mathcal{NS}o(\mathcal{S}_\rho)| = 0$.

$$|\eta I - \mathcal{NS}o(\mathcal{S}_\rho)| = \begin{vmatrix} \eta & -3\sqrt{2} & \cdots & -3\sqrt{2} & -\sqrt{10} & -\sqrt{10} & \cdots & -\sqrt{10} \\ -3\sqrt{2} & \eta & \cdots & -3\sqrt{2} & -\sqrt{10} & -\sqrt{10} & \cdots & -\sqrt{10} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -3\sqrt{2} & -3\sqrt{2} & \cdots & \eta & -\sqrt{10} & -\sqrt{10} & \cdots & -\sqrt{10} \\ -\sqrt{10} & -\sqrt{10} & \cdots & -\sqrt{10} & \eta & -\sqrt{2} & \cdots & -\sqrt{2} \\ -\sqrt{10} & -\sqrt{10} & \cdots & -\sqrt{10} & -\sqrt{2} & \eta & \cdots & -\sqrt{2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -\sqrt{10} & -\sqrt{10} & \cdots & -\sqrt{10} & -\sqrt{2} & -\sqrt{2} & \cdots & \eta \end{vmatrix}$$

From Lemma 1.1 the above determinant can be written as

$$|\eta I - \mathcal{NS}o(\mathcal{S}_\rho)| = \begin{vmatrix} (\eta I - 3\sqrt{2}(J-I))_{\rho\times\rho} & -\sqrt{10}J_{\rho\times\rho} \\ -\sqrt{10}J_{\rho\times\rho} & (\eta I - \sqrt{2}(J-I))_{\rho\times\rho} \end{vmatrix}$$

Expanding further we get,

$$|\eta I - \mathcal{NS}o(\mathcal{S}_\rho)| = (\eta+3\sqrt{2})^{\rho-1}(\eta+\sqrt{2})^{\rho-1}\{[\eta-(\rho-1)3\sqrt{2}][\eta-(\rho-1)\sqrt{2}]-10\rho^2\}$$

$Spec(\mathcal{NS}o(\mathcal{S}_\rho))$

$$= \begin{cases} -3\sqrt{2} & (\rho-1)times \\ -\sqrt{2} & (\rho-1)times \\ [3\sqrt{2}(\rho-1)+\sqrt{2}(\rho-1) \pm \sqrt{[3\sqrt{2}(\rho-1)-\sqrt{2}(\rho-1)]^2+40\rho^2}]/2 & \end{cases}$$

In this,

$$\eta_1 = [4\sqrt{2}(\rho-1)+\sqrt{8(\rho-1)^2+40\rho^2}]/2$$

is the largest and only one positive eigenvalue.

Hence

$$E_{\mathcal{NS}o}(\mathcal{S}_\rho) = 2\eta_1 = 4\sqrt{2}(\rho-1)+2\sqrt{2[(\rho-1)^2+5\rho^2]}$$

□

Theorem 3.2. *Let $\overline{\mathcal{S}}_\rho$, $\rho \geq 3$ be a complement of sunlet graph $\mathcal{S}_\rho$ then*

$$\begin{aligned} E_{\mathcal{NS}o}(\overline{\mathcal{S}}_\rho) &= 2[\sqrt{2}(\rho-2)(\rho-1)+\sqrt{2}(\rho-1)^2 \\ &+ \sqrt{2[(\rho-2)(\rho-1)-(\rho-1)^2]^2+4[(\rho-2)^2+(\rho-1)^2]\rho^2}] \end{aligned}$$

Proof. $\overline{\mathcal{S}}_\rho$, $\rho \geq 3$ is a complement of sunlet graph $\mathcal{S}_\rho$ with 2ρ vertices then the characteristic polynomial is given as $|\eta I - \mathcal{NS}o(\overline{\mathcal{S}}_\rho)| = 0$.

$$|\eta I - \mathcal{NS}o(\overline{\mathcal{S}}_\rho)| = \begin{vmatrix} \eta & \cdots & -2\sqrt{2}(\rho-2) & -2\sqrt{(\rho-2)^2+(\rho-1)^2} & \cdots & -2\sqrt{(\rho-2)^2+(\rho-1)^2} \\ -2\sqrt{2}(\rho-2) & \cdots & -2\sqrt{2}(\rho-2) & -2\sqrt{(\rho-2)^2+(\rho-1)^2} & \cdots & -2\sqrt{(\rho-2)^2+(\rho-1)^2} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ -2\sqrt{2}(\rho-2) & \cdots & \eta & -2\sqrt{(\rho-2)^2+(\rho-1)^2} & \cdots & -2\sqrt{(\rho-2)^2+(\rho-1)^2} \\ -2\sqrt{(\rho-2)^2+(\rho-1)^2} & \cdots & -2\sqrt{(\rho-2)^2+(\rho-1)^2} & \eta & \cdots & -2\sqrt{2}(\rho-1) \\ -2\sqrt{(\rho-2)^2+(\rho-1)^2} & \cdots & -2\sqrt{(\rho-2)^2+(\rho-1)^2} & -2\sqrt{2}(\rho-1) & \cdots & -2\sqrt{2}(\rho-1) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ -2\sqrt{(\rho-2)^2+(\rho-1)^2} & \cdots & -2\sqrt{(\rho-2)^2+(\rho-1)^2} & -2\sqrt{2}(\rho-1) & \cdots & \eta \end{vmatrix}$$

From Lemma 1.1 the above determinant can be written as

$$|\eta I - \mathcal{NS}o(\overline{\mathcal{S}}_\rho)| = \begin{vmatrix} (\eta I - 2\sqrt{2}(\rho-2)(J-I))_{\rho\times\rho} & -2\sqrt{(\rho-2)^2+(\rho-1)^2}J_{\rho\times\rho} \\ -2\sqrt{(\rho-2)^2+(\rho-1)^2}J_{\rho\times\rho} & (\eta I - 2\sqrt{2}(\rho-1)(J-I))_{\rho\times\rho} \end{vmatrix}$$

Expanding further we get,

$$\begin{aligned} |\eta I - \mathcal{NS}o(\overline{\mathcal{S}}_\rho)| &= (\eta+2\sqrt{2}(\rho-2))^{\rho-1}(\eta+2\sqrt{2}(\rho-1))^{\rho-1} \\ &\quad \{[\eta-(\rho-1)2\sqrt{2}(\rho-2)][\eta-(\rho-1)2\sqrt{2}(\rho-1)]-4[(\rho-2)^2+(\rho-1)^2]\rho^2\} \end{aligned}$$

$Spec(\mathcal{NS}o(\overline{\mathcal{S}}_\rho))$

$$= \begin{cases} -2\sqrt{2}(\rho-2) & (\rho-1) times \\ -2\sqrt{2}(\rho-1) & (\rho-1) times \\ [2\sqrt{2}(\rho-2)(\rho-1)+2\sqrt{2}(\rho-1)^2 & \\ \pm\sqrt{[2\sqrt{2}(\rho-2)(\rho-1)-2\sqrt{2}(\rho-1)^2]^2+16[(\rho-2)^2+(\rho-1)^2]\rho^2}]/2 & \end{cases}$$

In this,

$$\begin{aligned} \eta_1 &= 2[\sqrt{2}(\rho-2)(\rho-1)+\sqrt{2}(\rho-1)^2 \\ &+ \sqrt{2[(\rho-2)(\rho-1)-(\rho-1)^2]^2+4[(\rho-2)^2+(\rho-1)^2]\rho^2}]/2 \end{aligned}$$

is the largest and only one positive eigenvalue.

Hence

$$\begin{aligned} E_{\mathcal{NS}o}(\overline{\mathcal{S}}_\rho)=2\eta_1 &= 2[\sqrt{2}(\rho-2)(\rho-1)+\sqrt{2}(\rho-1)^2 \\ &+ \sqrt{2[(\rho-2)(\rho-1)-(\rho-1)^2]^2+4[(\rho-2)^2+(\rho-1)^2]\rho^2}] \end{aligned}$$

□

Theorem 3.3. *Let $\mathcal{B}_\rho, \rho \geq 1$ be a book graph then*

$$E_{\mathcal{NS}o}(\mathcal{B}_\rho)=\sqrt{2}(\rho+1)+2\sqrt{2}(2\rho-1)+\sqrt{[\sqrt{2}(\rho+1)-2\sqrt{2}(2\rho-1)]^2+16\rho[(\rho+1)^2+4]}$$

Proof. $\mathcal{B}_\rho$, $\rho \geq 1$ is a book graph with $2\rho+2$ vertices then the characteristic polynomial is given as $|\eta I-\mathcal{NS}o(\mathcal{B}_\rho)|=0$.

$$|\eta I-\mathcal{NS}o(\mathcal{B}_\rho)|=\begin{vmatrix} \eta & -\sqrt{2}(\rho+1) & -\sqrt{(\rho+1)^2+4} & -\sqrt{(\rho+1)^2+4} & \cdots & -\sqrt{(\rho+1)^2+4} \\ -\sqrt{2}(\rho+1) & \eta & -\sqrt{(\rho+1)^2+4} & -\sqrt{(\rho+1)^2+4} & \cdots & -\sqrt{(\rho+1)^2+4} \\ -\sqrt{(\rho+1)^2+4} & -\sqrt{(\rho+1)^2+4} & \eta & -2\sqrt{2} & \cdots & -2\sqrt{2} \\ -\sqrt{(\rho+1)^2+4} & -\sqrt{(\rho+1)^2+4} & -2\sqrt{2} & \eta & \cdots & -2\sqrt{2} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -\sqrt{(\rho+1)^2+4} & -\sqrt{(\rho+1)^2+4} & -2\sqrt{2} & -2\sqrt{2} & \cdots & \eta \end{vmatrix}$$

From Lemma 1.1 the above determinant can be written as

$$|\eta I-\mathcal{NS}o(\mathcal{B}_\rho)|=\begin{vmatrix} (\eta I-\sqrt{2}(\rho+1))(J-I))_{2\times 2} & -\sqrt{(\rho+1)^2+4}J_{2\times 2\rho} \\ -\sqrt{(\rho+1)^2+4}J_{2\rho\times 2} & (\eta I-2\sqrt{2}(J-I))_{2\rho\times 2\rho} \end{vmatrix}$$

Expanding further we get,

$$\begin{aligned} |\eta I-\mathcal{NS}o(\mathcal{B}_\rho)| &= (\eta+\sqrt{2}(\rho+1))^{2-1}(\eta+2\sqrt{2})^{2\rho-1} \\ & \{[\eta-(2-1)\sqrt{2}(\rho+1))][\eta-(2\rho-1)2\sqrt{2}]-4\rho[(\rho+1)^2+4]\} \end{aligned}$$

$Spec(\mathcal{NS}o(\mathcal{B}_\rho))$

$$= \begin{cases} -\sqrt{2}(\rho+1) & 1 time \\ -2\sqrt{2} & (2\rho-1) times \\ [\sqrt{2}(\rho+1)+2\sqrt{2}(2\rho-1)\pm & \\ \sqrt{[\sqrt{2}(\rho+1)-2\sqrt{2}(2\rho-1)]^2+16\rho[(\rho+1)^2+4]}]/2 & \end{cases}$$

In this,

$$\eta_1 = [\sqrt{2}(\rho+1) + 2\sqrt{2}(2\rho-1) + \sqrt{[\sqrt{2}(\rho+1) - 2\sqrt{2}(2\rho-1)]^2 + 16\rho[(\rho+1)^2+4]}]/2$$

is the largest and only one positive eigenvalue.

Hence

$$\begin{aligned} E_{\mathcal{NS}o}(\mathcal{B}_\rho) &= 2\eta_1 = \sqrt{2}(\rho+1) + 2\sqrt{2}(2\rho-1) \\ &+ \sqrt{[\sqrt{2}(\rho+1) - 2\sqrt{2}(2\rho-1)]^2 + 16\rho[(\rho+1)^2+4]} \end{aligned}$$

□

Theorem 3.4. *Let $\overline{\mathcal{B}}_\rho$, $\rho \geq 1$ be a complement of book graph $\mathcal{B}_\rho$ then*

$$E_{\mathcal{NS}o}(\overline{\mathcal{B}}_\rho) = \sqrt{2}\rho + \sqrt{2}(2\rho-1)^2 + \sqrt{[\sqrt{2}\rho - \sqrt{2}(2\rho-1)^2]^2 + 16\rho[(2\rho-1)^2+\rho^2]}$$

Proof. $\overline{\mathcal{B}}_\rho$, $\rho \geq 1$ is a complement of book graph $\mathcal{B}_\rho$ with $2\rho+2$ vertices then the characteristic polynomial is given as $|\eta I - \mathcal{NS}o(\overline{\mathcal{B}}_\rho)| = 0$.

$$|\eta I - \mathcal{NS}o(\overline{\mathcal{B}}_\rho)| = \begin{vmatrix} \eta & -\sqrt{2}\rho & -\sqrt{(2\rho-1)^2+\rho^2} & -\sqrt{(2\rho-1)^2+\rho^2} & \cdots & -\sqrt{(2\rho-1)^2+\rho^2} \\ -\sqrt{2}\rho & \eta & -\sqrt{(2\rho-1)^2+\rho^2} & -\sqrt{(2\rho-1)^2+\rho^2} & \cdots & -\sqrt{(2\rho-1)^2+\rho^2} \\ -\sqrt{(2\rho-1)^2+\rho^2} & -\sqrt{(2\rho-1)^2+\rho^2} & \eta & -\sqrt{2}(2\rho-1) & \cdots & -\sqrt{2}(2\rho-1) \\ -\sqrt{(2\rho-1)^2+\rho^2} & -\sqrt{(2\rho-1)^2+\rho^2} & -\sqrt{2}(2\rho-1) & \eta & \cdots & -\sqrt{2}(2\rho-1) \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -\sqrt{(2\rho-1)^2+\rho^2} & -\sqrt{(2\rho-1)^2+\rho^2} & -\sqrt{2}(2\rho-1) & -\sqrt{2}(2\rho-1) & \cdots & \eta \end{vmatrix}$$

From Lemma 1.1 the above determinant can be written as

$$|\eta I - \mathcal{NS}o(\overline{\mathcal{B}}_\rho)| = \begin{vmatrix} (\eta I - \sqrt{2}\rho)(J-I))_{2\times 2} & -\sqrt{(2\rho-1)^2+\rho^2}J_{2\times 2\rho} \\ -\sqrt{(2\rho-1)^2+\rho^2}J_{2\rho\times 2} & (\eta I - \sqrt{2}(2\rho-1)(J-I))_{2\rho\times 2\rho} \end{vmatrix}$$

Expanding further we get,

$$\begin{aligned} |\eta I - \mathcal{NS}o(\overline{\mathcal{B}}_\rho)| &= (\eta + \sqrt{2}\rho)^{2-1}(\eta + \sqrt{2}(2\rho-1))^{2\rho-1} \\ &\quad \{[\eta - (2-1)\sqrt{2}\rho)][\eta - (2\rho-1)\sqrt{2}(2\rho-1)] - 4\rho[(2\rho-1)^2+\rho^2]\} \end{aligned}$$

$Spec(\mathcal{NS}o(\overline{\mathcal{B}}_\rho))$

$$= \begin{cases} -\sqrt{2}\rho & 1 time \\ -\sqrt{2}(2\rho-1) & (2\rho-1) times \\ [\sqrt{2}\rho + \sqrt{2}(2\rho-1)^2 \pm \sqrt{[\sqrt{2}\rho - \sqrt{2}(2\rho-1)^2]^2 + 16\rho[(2\rho-1)^2+\rho^2]}]/2 & \end{cases}$$

In this,

$$\eta_1 = [\sqrt{2}\rho + \sqrt{2}(2\rho-1)^2 + \sqrt{[\sqrt{2}\rho - \sqrt{2}(2\rho-1)^2]^2 + 16\rho[(2\rho-1)^2+\rho^2]}]/2$$

is the largest and only one positive eigenvalue.

Hence

$$E_{\mathcal{NS}o}(\overline{\mathcal{B}}_\rho) = \sqrt{2}\rho + \sqrt{2}(2\rho-1)^2 + \sqrt{[\sqrt{2}\rho - \sqrt{2}(2\rho-1)^2]^2 + 16\rho[(2\rho-1)^2+\rho^2]}$$

□

Theorem 3.5. *Let $\mathcal{PS}_\rho, \rho \geq 2$ be a pentagonal snake graph then*

$$E_{\mathcal{NS}o}(\mathcal{PS}_\rho) = 2\sqrt{2}(3\rho-2)+4\sqrt{2}(\rho-3)+\sqrt{[2\sqrt{2}(3\rho-2)-4\sqrt{2}(\rho-3)]^2+80(3\rho-1)(\rho-2)}$$

Proof. $\mathcal{PS}_\rho$, $\rho \geq 2$ is a pentagonal snake graph with $4\rho-3$ vertices then the characteristic polynomial is given as $|\eta I - \mathcal{NS}o(\mathcal{PS}_\rho)| = 0$.

$$|\eta I - \mathcal{NS}o(\mathcal{PS}_\rho)| = \begin{vmatrix} \eta & -2\sqrt{2} & \cdots & -2\sqrt{2} & -2\sqrt{5} & -2\sqrt{5} & \cdots & -2\sqrt{5} \\ -2\sqrt{2} & \eta & \cdots & -2\sqrt{2} & -2\sqrt{5} & -2\sqrt{5} & \cdots & -2\sqrt{5} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -2\sqrt{2} & -2\sqrt{2} & \cdots & \eta & -2\sqrt{5} & -2\sqrt{5} & \cdots & -2\sqrt{5} \\ -2\sqrt{5} & -2\sqrt{5} & \cdots & -2\sqrt{5} & \eta & -4\sqrt{2} & \cdots & -4\sqrt{2} \\ -2\sqrt{5} & -2\sqrt{5} & \cdots & -2\sqrt{5} & -4\sqrt{2} & \eta & \cdots & -4\sqrt{2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -2\sqrt{5} & -2\sqrt{5} & \cdots & -2\sqrt{5} & -4\sqrt{2} & -4\sqrt{2} & \cdots & \eta \end{vmatrix}$$

From Lemma 1.1 the above determinant can be written as

$$|\eta I - \mathcal{NS}o(\mathcal{PS}_\rho)| = \begin{vmatrix} (\eta I - 2\sqrt{2}(J-I))_{3\rho-1\times 3\rho-1} & -2\sqrt{5}J_{3\rho-1\times\rho-2} \\ -2\sqrt{5}J_{\rho-2\times 3\rho-1} & (\eta I - 4\sqrt{2}(J-I))_{\rho-2\times\rho-2} \end{vmatrix}$$

Expanding further we get,

$$\begin{aligned} |\eta I - \mathcal{NS}o(\mathcal{PS}_\rho)| &= (\eta+2\sqrt{2})^{3\rho-1-1}(\eta+4\sqrt{2})^{\rho-2-1} \\ &= \{[\eta-(3\rho-1-1)2\sqrt{2}][\eta-(\rho-2-1)4\sqrt{2}]-20(3\rho-1)(\rho-2)\} \\ |\eta I - \mathcal{NS}o(\mathcal{PS}_\rho)| &= (\eta+2\sqrt{2})^{3\rho-2}(\eta+4\sqrt{2})^{\rho-3} \\ &= \{[\eta-(3\rho-2)2\sqrt{2}][\eta-(\rho-3)4\sqrt{2}]-20(3\rho-1)(\rho-2)\} \end{aligned}$$

$Spec(\mathcal{NS}o(\mathcal{PS}_\rho))$

$$= \begin{cases} -2\sqrt{2} & (3\rho-2) times \\ -4\sqrt{2} & (\rho-3) times \\ [2\sqrt{2}(3\rho-2)+4\sqrt{2}(\rho-3)\pm & \\ \sqrt{[2\sqrt{2}(3\rho-2)-4\sqrt{2}(\rho-3)]^2+80(3\rho-1)(\rho-2)}]/2 & \end{cases}$$

In this,

$$\eta_1 = [2\sqrt{2}(3\rho-2)+4\sqrt{2}(\rho-3)+\sqrt{[2\sqrt{2}(3\rho-2)-4\sqrt{2}(\rho-3)]^2+80(3\rho-1)(\rho-2)}]/2$$

is the largest and only one positive eigenvalue.

Hence

$$\begin{aligned} E_{\mathcal{NS}o}(\mathcal{PS}_\rho) &= 2\eta_1 = 2\sqrt{2}(3\rho-2)+4\sqrt{2}(\rho-3) \\ &+ \sqrt{[2\sqrt{2}(3\rho-2)-4\sqrt{2}(\rho-3)]^2+80(3\rho-1)(\rho-2)} \end{aligned}$$

□

Theorem 3.6. *Let $\overline{\mathcal{PS}}_\rho$, $\rho \geq 2$ be a complement pentagonal snake graph then*

$$\begin{aligned} E_{\mathcal{NS}o}(\overline{\mathcal{PS}}_\rho) &= 2\sqrt{2}(2\rho-3)(3\rho-2)+4\sqrt{2}(\rho-2)(\rho-3) \\ &\pm [[2\sqrt{2}(2\rho-3)(3\rho-2)-4\sqrt{2}(\rho-2)(\rho-3)]^2 \\ &+ 16[(2\rho-3)^2+4(\rho-2)^2](3\rho-1)(\rho-2)]^{1/2} \end{aligned}$$

Proof. $\overline{\mathcal{PS}}_\rho$, $\rho \geq 2$ is a complement of pentagonal snake graph $\mathcal{PS}_\rho$ with $4\rho-3$ vertices then the characteristic polynomial is given as $|\eta I - \mathcal{NS}o(\overline{\mathcal{PS}}_\rho)| = 0$.

$|\eta I - \mathcal{NS}o(\overline{\mathcal{PS}}_\rho)| =$

$$\begin{vmatrix} \eta & \cdots & -2\sqrt{2}(2\rho-3) & -2\sqrt{(2\rho-3)^2+4(\rho-2)^2} & \cdots & -2\sqrt{(2\rho-3)^2+4(\rho-2)^2} \\ -2\sqrt{2}(2\rho-3) & \cdots & -2\sqrt{2}(2\rho-3) & -2\sqrt{(2\rho-3)^2+4(\rho-2)^2} & \cdots & -2\sqrt{(2\rho-3)^2+4(\rho-2)^2} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ -2\sqrt{2}(2\rho-3) & \cdots & \eta & -2\sqrt{(2\rho-3)^2+4(\rho-2)^2} & \cdots & -2\sqrt{(2\rho-3)^2+4(\rho-2)^2} \\ -2\sqrt{(2\rho-3)^2+4(\rho-2)^2} & \cdots & -2\sqrt{(2\rho-3)^2+4(\rho-2)^2} & \eta & \cdots & -4\sqrt{2}(\rho-2) \\ -2\sqrt{(2\rho-3)^2+4(\rho-2)^2} & \cdots & -2\sqrt{(2\rho-3)^2+4(\rho-2)^2} & -4\sqrt{2}(\rho-2) & \cdots & -4\sqrt{2}(\rho-2) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ -2\sqrt{(2\rho-3)^2+4(\rho-2)^2} & \cdots & -2\sqrt{(2\rho-3)^2+4(\rho-2)^2} & -4\sqrt{2}(\rho-2) & \cdots & \eta \end{vmatrix}$$

From Lemma 1.1 the above determinant can be written as

$$|\eta I - \mathcal{NS}o(\overline{\mathcal{PS}}_\rho)| = \begin{vmatrix} (\eta I - 2\sqrt{2}(2\rho-3)(J-I))_{3\rho-1\times 3\rho-1} & -2\sqrt{(2\rho-3)^2+4(\rho-2)^2}J_{3\rho-1\times\rho-2} \\ -2\sqrt{(2\rho-3)^2+4(\rho-2)^2}J_{\rho-2\times 3\rho-1} & (\eta I - 4\sqrt{2}(\rho-2)(J-I))_{\rho-2\times\rho-2} \end{vmatrix}$$

Expanding further we get,

$$\begin{aligned} |\eta I - \mathcal{NS}o(\overline{\mathcal{PS}}_\rho)| &= (\eta+2\sqrt{2}(2\rho-3))^{3\rho-1-1}(\eta+4\sqrt{2}(\rho-2))^{\rho-2-1}\{[\eta-(3\rho-1-1)2\sqrt{2}(2\rho-3)] \\ &= [\eta-(\rho-2-1)4\sqrt{2}(\rho-2)] - 4[(2\rho-3)^2+4(\rho-2)^2](3\rho-1)(\rho-2)\} \\ |\eta I - \mathcal{NS}o(\mathcal{PS}_\rho)| &= (\eta+2\sqrt{2}(2\rho-3))^{3\rho-2}(\eta+4\sqrt{2}(\rho-2))^{\rho-3}\{[\eta-(3\rho-2)2\sqrt{2}(2\rho-3)] \\ &= [\eta-(\rho-3)4\sqrt{2}(\rho-2)] - 4[(2\rho-3)^2+4(\rho-2)^2](3\rho-1)(\rho-2)\} \end{aligned}$$

$Spec(\mathcal{NS}o(\overline{\mathcal{PS}}_\rho))$

$$= \begin{cases} -2\sqrt{2}(2\rho-3) & (3\rho-2) times \\ -4\sqrt{2}(\rho-2) & (\rho-3) times \\ [2\sqrt{2}(2\rho-3)(3\rho-2)+4\sqrt{2}(\rho-2)(\rho-3)\pm & \\ [[2\sqrt{2}(2\rho-3)(3\rho-2)-4\sqrt{2}(\rho-2)(\rho-3)]^2 & \\ +16[(2\rho-3)^2+4(\rho-2)^2](3\rho-1)(\rho-2)]^{1/2}]/2 & \end{cases}$$

In this,

$$\begin{aligned} \eta_1 &= [2\sqrt{2}(2\rho-3)(3\rho-2)+4\sqrt{2}(\rho-2)(\rho-3) \\ &\pm \sqrt{[2\sqrt{2}(2\rho-3)(3\rho-2)-4\sqrt{2}(\rho-2)(\rho-3)]^2+16[(2\rho-3)^2+4(\rho-2)^2](3\rho-1)(\rho-2)}]/2 \end{aligned}$$

is the largest and only one positive eigenvalue.

Hence

$$\begin{aligned} E_{\mathcal{NS}o}(\overline{\mathcal{PS}}_\rho) &= 2\eta_1 = 2\sqrt{2}(2\rho-3)(3\rho-2)+4\sqrt{2}(\rho-2)(\rho-3) \\ &\pm [[2\sqrt{2}(2\rho-3)(3\rho-2)-4\sqrt{2}(\rho-2)(\rho-3)]^2 \end{aligned}$$

$$+ \quad 16[(2\rho-3)^2+4(\rho-2)^2](3\rho-1)(\rho-2)]^{1/2}$$

□

Theorem 3.7. *Let $\mathcal{W}_\rho^3$, $\rho \geq 3$ be a windmill graph then*

$$E_{\mathcal{NS}o}(\mathcal{W}_\rho^3) = (\rho-1)[\sqrt{2}(3\rho-4)+\sqrt{2[(3\rho-4)^2+60(\rho-1)}]$$

Proof. $\mathcal{W}_\rho^3$, $\rho \geq 3$ is a windmill graph with $3\rho-2$ vertices then

$$(\mathcal{NS}o(\mathcal{W}_\rho^3)) = \begin{vmatrix} 0 & \sqrt{10}(\rho-1) & \sqrt{10}(\rho-1) & \sqrt{10}(\rho-1) & \cdots & \sqrt{10}(\rho-1) \\ \sqrt{10}(\rho-1) & 0 & \sqrt{2}(\rho-1) & \sqrt{2}(\rho-1) & \cdots & \sqrt{2}(\rho-1) \\ \sqrt{10}(\rho-1) & \sqrt{2}(\rho-1) & 0 & \sqrt{2}(\rho-1) & \cdots & \sqrt{2}(\rho-1) \\ \sqrt{10}(\rho-1) & \sqrt{2}(\rho-1) & \sqrt{2}(\rho-1) & 0 & \cdots & \sqrt{2}(\rho-1) \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \sqrt{10}(\rho-1) & \sqrt{2}(\rho-1) & \sqrt{2}(\rho-1) & \sqrt{2}(\rho-1) & \cdots & 0 \end{vmatrix}$$

From Lemma 1.2 the characteristic polynomial is given by

$$\begin{aligned} |\eta I - \mathcal{NS}o(\mathcal{W}_\rho^3)| &= (\eta+\sqrt{2}(\rho-1))^{3\rho-4}[\eta^2-(3\rho-4)\sqrt{2}(\rho-1)\eta-(3\rho-3)(\sqrt{10}(\rho-1))^2] \\ &= (\eta+\sqrt{2}(\rho-1))^{3\rho-4}[\eta^2-\sqrt{10}(3\rho-4)(\rho-1)\eta-30(\rho-1)^3] \end{aligned}$$

Also,

$$Spectra = \begin{cases} -\sqrt{2}(\rho-1) & (3\rho-4) times \\ [(3\rho-4)\sqrt{2}(\rho-1) \pm \sqrt{[2(3\rho-4)^2(\rho-1)^2]+120(\rho-1)^3}]/2 \end{cases}$$

In this,

$$\eta_1 = (\rho-1)[\sqrt{2}(3\rho-4)+\sqrt{2[(3\rho-4)^2+60(\rho-1)}]/2$$

is the largest and only one positive eigenvalue.

Hence

$$E_{\mathcal{NS}o}(\mathcal{W}_\rho^3) \quad = \quad 2\eta_1 = (\rho-1)[\sqrt{2}(3\rho-4)+\sqrt{2[(3\rho-4)^2+60(\rho-1)}]$$

□

Theorem 3.8. *Let $\overline{\mathcal{W}}_\rho^3$, $\rho \geq 3$ be the complement of windmill graph $\mathcal{W}_\rho^3$ then*

$$E_{\mathcal{NS}o}(\overline{\mathcal{W}}_\rho^3) = 2(\rho-1)[\sqrt{2}(3\rho-4)+\sqrt{2[(3\rho-4)^2+6(\rho-1)}]$$

Proof. $\overline{\mathcal{W}}_\rho^3$, $\rho \geq 3$ is a complement of windmill graph $\mathcal{W}_\rho^3$ with $3\rho-2$ vertices then

$$(\mathcal{NS}o(\overline{\mathcal{W}}_\rho^3)) = \begin{vmatrix} 0 & 2(\rho-1) & 2(\rho-1) & 2(\rho-1) & \cdots & 2(\rho-1) \\ 2(\rho-1) & 0 & 2\sqrt{2}(\rho-1) & 2\sqrt{2}(\rho-1) & \cdots & 2\sqrt{2}(\rho-1) \\ 2(\rho-1) & 2\sqrt{2}(\rho-1) & 0 & 2\sqrt{2}(\rho-1) & \cdots & 2\sqrt{2}(\rho-1) \\ 2(\rho-1) & 2\sqrt{2}(\rho-1) & 2\sqrt{2}(\rho-1) & 0 & \cdots & 2\sqrt{2}(\rho-1) \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 2(\rho-1) & 2\sqrt{2}(\rho-1) & 2\sqrt{2}(\rho-1) & 2\sqrt{2}(\rho-1) & \cdots & 0 \end{vmatrix}$$

From Lemma 1.2 the characteristic polynomial is given by

$$\begin{aligned}|\eta I - \mathcal{NS}o(\overline{\mathcal{W}}_\rho^3)| &= (\eta + 2\sqrt{2}(\rho-1))^{3\rho-4}[\eta^2 - (3\rho-4)2\sqrt{2}(\rho-1)\eta - (3\rho-3)(2(\rho-1))^2] \\ &= (\eta + 2\sqrt{2}(\rho-1))^{3\rho-4}[\eta^2 - 2\sqrt{2}(3\rho-4)(\rho-1)\eta - 12(\rho-1)^3]\end{aligned}$$

Also,

$$Spectra = \begin{cases} -2\sqrt{2}(\rho-1) & (3\rho-4) times \\ [(3\rho-4)2\sqrt{2}(\rho-1) \pm \sqrt{[8(3\rho-4)^2(\rho-1)^2 + 48(\rho-1)^3}]/2 & \end{cases}$$

In this,

$$\eta_1 = 2(\rho-1)[\sqrt{2}(3\rho-4) + \sqrt{2[(3\rho-4)^2 + 6(\rho-1)}]/2$$

is the largest and only one positive eigenvalue.

Hence

$$E_{\mathcal{NS}o}(\overline{\mathcal{W}}_\rho^3) = 2\eta_1 = 2(\rho-1)[\sqrt{2}(3\rho-4) + \sqrt{2[(3\rho-4)^2 + 6(\rho-1)}]$$

□

Theorem 3.9. *Let $\mathcal{F}_\rho$, $\rho \geq 1$ be a friendship graph then*

$$E_{\mathcal{NS}o}(\mathcal{F}_\rho) = 2[\sqrt{2}(2\rho-1) + \sqrt{2[(2\rho-1)^2 + 4\rho(\rho^2+1)}]$$

Proof. $\mathcal{F}_\rho$, $\rho \geq 1$ is a friendship graph with $2\rho+1$ vertices then

$$(\mathcal{NS}o(\mathcal{F}_\rho)) = \begin{vmatrix} 0 & 2\sqrt{\rho^2+1} & 2\sqrt{\rho^2+1} & 2\sqrt{\rho^2+1} & \cdots & 2\sqrt{\rho^2+1} \\ 2\sqrt{\rho^2+1} & 0 & 2\sqrt{2} & 2\sqrt{2} & \cdots & 2\sqrt{2} \\ 2\sqrt{\rho^2+1} & 2\sqrt{2} & 0 & 2\sqrt{2} & \cdots & 2\sqrt{2} \\ 2\sqrt{\rho^2+1} & 2\sqrt{2} & 2\sqrt{2} & 0 & \cdots & 2\sqrt{2} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 2\sqrt{\rho^2+1} & 2\sqrt{2} & 2\sqrt{2} & 2\sqrt{2} & \cdots & 0 \end{vmatrix}$$

From Lemma 1.2 the characteristic polynomial is given by

$$\begin{aligned}|\eta I - \mathcal{NS}o(\mathcal{F}_\rho)| &= (\eta + 2\sqrt{2})^{2\rho-1}[\eta^2 - (2\rho-1)2\sqrt{2}\eta - (2\rho)(2\sqrt{\rho^2+1})^2] \\ &= (\eta + 2\sqrt{2})^{2\rho-1}[\eta^2 - 2\sqrt{2}(2\rho-1)\eta - 8\rho(\rho^2+1)]\end{aligned}$$

Also,

$$Spectra = \begin{cases} -2\sqrt{2} & (2\rho-1) times \\ [(2\rho-1)2\sqrt{2} \pm \sqrt{[8(2\rho-1)^2 + 32\rho(\rho^2+1)}]/2 & \end{cases}$$

In this,

$$\eta_1 = 2[\sqrt{2}(2\rho-1) + \sqrt{2[(2\rho-1)^2 + 4\rho(\rho^2+1)}]/2$$

is the largest and only one positive eigenvalue.

Hence

$$E_{\mathcal{NS}o}(\mathcal{F}_\rho) = 2\eta_1 = 2[\sqrt{2}(2\rho-1) + \sqrt{2[(2\rho-1)^2 + 4\rho(\rho^2+1)}]$$

□

Theorem 3.10. *Let $\overline{\mathcal{F}}_\rho$, $\rho \geq 1$ be the complement of friendship graph $\mathcal{F}_\rho$ then*

$$E_{\mathcal{NS}o}(\overline{\mathcal{F}}_\rho) = 2(\rho-1)[\sqrt{2}(2\rho-1) + \sqrt{2[(2\rho-1)^2+4\rho}]$$

Proof. $\overline{\mathcal{F}}_\rho$, $\rho \geq 1$ is a complement of friendship graph $\mathcal{F}_\rho$ with $2\rho+1$ vertices then

$$(\mathcal{NS}o(\overline{\mathcal{F}}_\rho)) = \begin{vmatrix} 0 & 2(\rho-1) & 2(\rho-1) & 2(\rho-1) & \cdots & 2(\rho-1) \\ 2(\rho-1) & 0 & 2\sqrt{2}(\rho-1) & 2\sqrt{2}(\rho-1) & \cdots & 2\sqrt{2}(\rho-1) \\ 2(\rho-1) & 2\sqrt{2}(\rho-1) & 0 & 2\sqrt{2}(\rho-1) & \cdots & 2\sqrt{2}(\rho-1) \\ 2(\rho-1) & 2\sqrt{2}(\rho-1) & 2\sqrt{2}(\rho-1) & 0 & \cdots & 2\sqrt{2}(\rho-1) \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 2(\rho-1) & 2\sqrt{2}(\rho-1) & 2\sqrt{2}(\rho-1) & 2\sqrt{2}(\rho-1) & \cdots & 0 \end{vmatrix}$$

From Lemma 1.2 the characteristic polynomial is given by

$$\begin{aligned} |\eta I - \mathcal{NS}o(\overline{\mathcal{F}}_\rho)| &= (\eta + 2\sqrt{2}(\rho-1))^{2\rho-1}[\eta^2 - (2\rho-1)2\sqrt{2}(\rho-1)\eta - (2\rho)(2(\rho-1))^2] \\ &= (\eta + 2\sqrt{2}(\rho-1))^{2\rho-1}[\eta^2 - 2\sqrt{2}(2\rho-1)(\rho-1)\eta - 8\rho(\rho-1)^2] \end{aligned}$$

Also,

$$Spectra = \begin{cases} -2\sqrt{2}(\rho-1) & (2\rho-1) times \\ [(2\rho-1)2\sqrt{2}(\rho-1) \pm \sqrt{[8(2\rho-1)^2(\rho-1)^2 + 32\rho(\rho-1)^2}]/2 & \end{cases}$$

In this,

$$\eta_1 = 2(\rho-1)[\sqrt{2}(2\rho-1) + \sqrt{2[(2\rho-1)^2+4\rho}]/2$$

is the largest and only one positive eigenvalue.

Hence

$$E_{\mathcal{NS}o}(\overline{\mathcal{F}}_\rho) \quad = \quad 2\eta_1 = 2(\rho-1)[\sqrt{2}(2\rho-1) + \sqrt{2[(2\rho-1)^2+4\rho}]$$

□

4. Conclusion

In this paper, a new variant Sombor matrix $\mathcal{NS}o(\zeta)$ is defined as, if $i \neq j$ its (i,j)-entry is equal to $\sqrt{\varsigma_i^2 + \varsigma_j^2}$ and 0 in the either case, where ς_i represents the degree of i^{th} vertex. $E_{\mathcal{NS}o}(\zeta)$ is obtained for some standard class of graphs. Also the spectrum and $E_{\mathcal{NS}o}(\zeta)$ is obtained for some families of graphs and its complements.

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